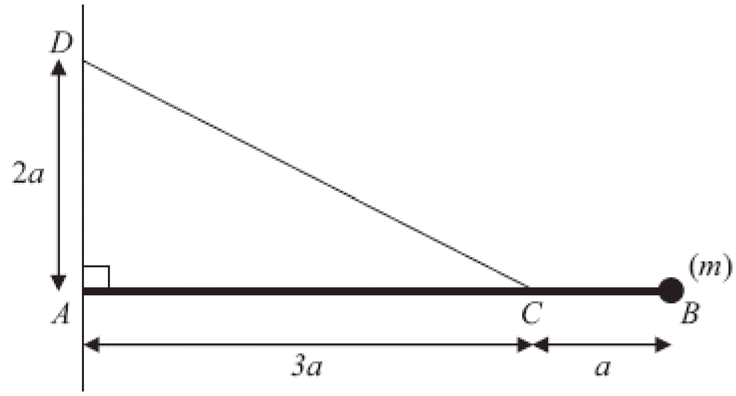
1.



The diagram above shows a uniform rod AB of mass m and length $4a$. The end A of the rod is freely hinged to a point on a vertical wall. A particle of mass m is attached to the rod at B. One end of a light inextensible string is attached to the rod at C, where $AC = 3a$. The other end of the string is attached to the wall at D, where $AD = 2a$ and D is vertically above A. The rod rests horizontally in equilibrium in a vertical plane perpendicular to the wall and the tension in the string is T.

(a) Show that $T = mg\sqrt{13}$.

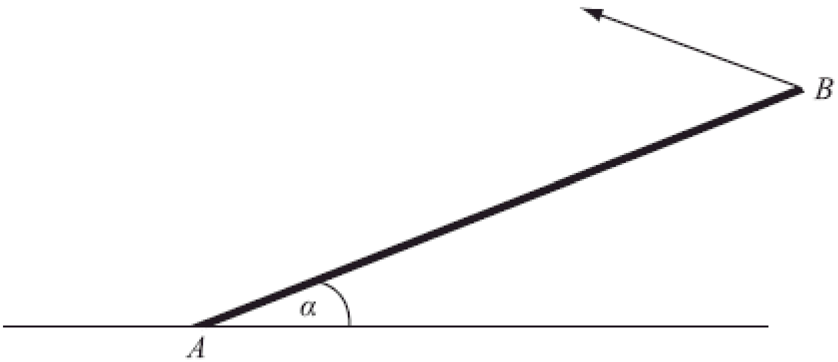
(5)

The particle of mass m at B is removed from the rod and replaced by a particle of mass M which is attached to the rod at B. The string breaks if the tension exceeds $2mg\sqrt{13}$. Given that the string does not break,

(b) show that $M \leq \frac{5}{2}m$.

(3)
(Total 8 marks)

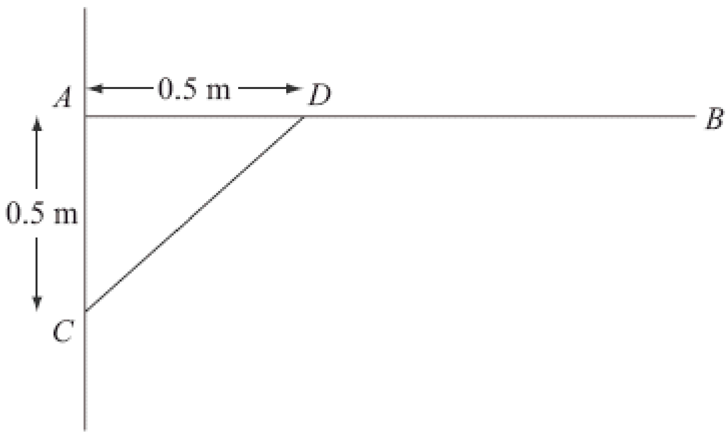
2.



A uniform rod *AB*, of mass 20 kg and length 4 m, rests with one end *A* on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where tan $\alpha = \frac{3}{4}$, by a force acting at *B*, as shown in Figure 2. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.5. Find the magnitude of the normal reaction of the ground on the rod at *A*.

(Total 7 marks)

3.



A uniform rod *AB*, of length 1.5 m and mass 3 kg, is smoothly hinged to a vertical wall at *A*. The rod is held in equilibrium in a horizontal position by a light strut *CD* as shown in the diagram above. The rod and the strut lie in the same vertical plane, which is perpendicular to the wall. The end *C* of the strut is freely jointed to the wall at a point 0.5 m vertically below *A*. The end *D* is freely joined to the rod so that *AD* is 0.5 m.

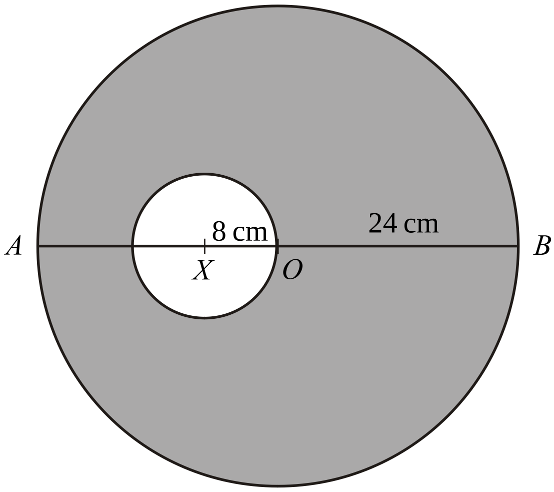
(a) Find the thrust in *CD*.

(4)

(b) Find the magnitude and direction of the force exerted on the rod AB at A.

(7)
(Total 11 marks)

4.



The diagram above shows a template T made by removing a circular disc, of centre X and radius 8 cm, from a uniform circular lamina, of centre O and radius 24 cm. The point X lies on the diameter AOB of the lamina and AX = 16 cm. The centre of mass of T is at the point G.

(a) Find AG.

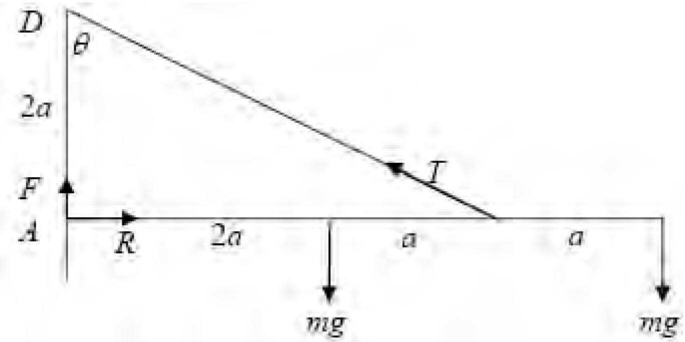
(6)

The template T is free to rotate about a smooth fixed horizontal axis, perpendicular to the plane of T, which passes through the mid-point of OB. A small stud of mass $\frac{1}{4}m$ is fixed at B, and T and the stud are in equilibrium with AB horizontal. Modelling the stud as a particle,

(b) find the mass of T in terms of m.

(4)
(Total 10 marks)

1. (a)



M(A) $3a \times T\cos\theta = 2amg + 4amg$ M1 A1 A1

$\cos\theta = \left(\dfrac{2}{\sqrt{9+4}} = \right)\dfrac{2}{\sqrt{13}}$ B1

$\dfrac{6}{\sqrt{13}}T = 6mg$

$T = mg\sqrt{13}$ * A1 5

(b) $3a \times T \times \cos\theta = 2amg + 4aMg$ M1

$T = \dfrac{(2mg+4Mg)}{6}\sqrt{13} \le 2mg13$ A1

$mg + 2MG \le 6mg$

$M \le \dfrac{5}{2}m$ * cso A1 3

[8]

2. $m(B)$: $R\times 4\cos\alpha = F \times 4\sin\alpha + 20g \times 2\cos\alpha$ M1 A2

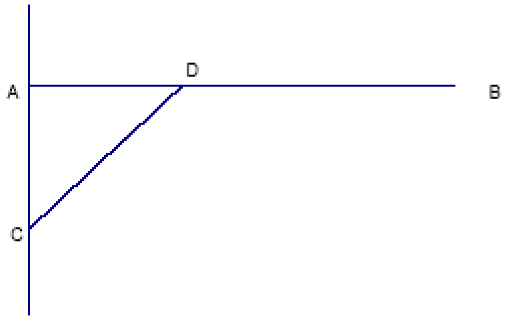
Use of $F = \dfrac{1}{2}R$ M1

Use of correct trig ratios B1

R = 160N or 157N DM1 A1

[7]

3. (a)



Taking moments about A:

$3g \times 0.75 = \frac{T}{\sqrt{2}} \times 0.5$ M1A1A1

$T = 3\sqrt{2}g \times \frac{7.5}{5} = \frac{9\sqrt{2}g}{2} (= 62.4N)$ A1 4

(b) $\leftarrow \pm H = \frac{T}{\sqrt{2}} \left(= \frac{9g}{2} \approx 44.1N \right)$ B1

$\uparrow \pm V + \frac{T}{\sqrt{2}} = 3g \left(\Rightarrow V = 3g - \frac{9g}{2} = \frac{-3g}{2} \approx -14.7N \right)$ M1A1

$|R| = \sqrt{81+9} \times \frac{g}{2} \approx 46.5(N)$ M1A1

at angle $\tan^{-1}\frac{1}{3} =$ 18.4° (0.322 radians) below the line of BA M1A1 7

161.6° (2.82 radians) below the line of AB

(108.4° or 1.89 radians to upward vertical)

[11]

4. (a)

	Large	Small	Template		
Mass ratios	24^2	8^2,	512	anything in ratio 9: 1 : 8	B1,B1ft
	(c.1810	*c.200*	*c.1610)*		

M(A) $9 \times 24 = 16 \times 1 + 8\bar{x}$ M1*A1

$\bar{x} = 25$ (cm) exact DM1*A1 6

(b)	M(axis) 1LM = 12 × $\frac{1}{4}m$	ft their $\bar{x}$	M1†A1ft	
	$\left((36-\bar{x})M = 12\times\frac{1}{4}m\right)$			
	$M = \frac{3}{11}m$ (o.e.e.)		DM1†A1	4

[10]

1. There were many correct responses to this question, some considerably more concise than others.

In part (a) many candidates took the direct route of the mark scheme, and most dealt confidently with the exact trig ratio. There were several who had initially made a false start, resolving vertically and horizontally and ignoring all or part of the reaction at the hinge, but they often went on to score full marks by later taking moments about A correctly. Some did not seem to understand the significance of requiring an exact answer and obtained the given answer in surd form from a decimal value of sin 33.69°.

In part (b) most candidates learned from their experience in (a) and started by taking moments about A. A minority tried several alternative options before deciding to take moments about A. Some candidates did not deal appropriately with the inequality, either by including it when taking moments or by simply inserting it in the final line.

2 There were very few correct solutions to this question that did not involve taking moments about B. Many candidates seemed to assume that the lack of any information about the direction of the force at B was an omission rather than a hint on how to proceed.
Those candidates who started by taking moments about B usually reached the required answer without difficulty. The most common errors involved confusion between sine and cosine, and inappropriate accuracy in the final answer after using a decimal approximation for g.
Alternative methods involving the force at B rarely produced a complete solution. Many candidates assumed that the direction of this force involved the angle α, thus simplifying the algebraic manipulation of their force and moment equations. Those who introduced an unknown angle usually struggled to reach a valid answer, although a handful of concise, correct solutions were seen.

3. In part (a) candidates who realised the thrust in the strut acted in the direction CD were generally successful in finding its magnitude. The overwhelming majority applied the most simple method of taking moments about A, although much longer alternative methods were often seen. A significant number, however, appeared to be confused by the use of the word thrust and many took this to be a vertical force acting at D.

For part (b) many candidates were able to find the horizontal and vertical components of the reaction at A and then the correct magnitude of this force. Most went on to find a direction but a significant number were unable to describe this direction properly. It was surprising to see relatively few diagrams and yet a diagram would have shown direction clearly.

The confusion over the direction of the thrust led to many errors in the vertical component of the reaction. Some candidates falsely assumed that the reaction at A acted vertically/horizontally. The candidates who took the force at D to be vertical were unable to complete this part as there was no horizontal force present.

4. Many candidates reached for calculators in this question – it was unusual to see the simplified form (9 : 1 : 8) for the ratio of areas. Despite this, many solved the problem successfully. The most common error was in failing to subtract the area of the disc removed in the moments equation or in failing to subtract the moment of the disc removed. Candidates did not always choose to take moments about A, but many correct solutions were seen. A minority of candidates were either searching the formula booklet for inspiration or confused by more advanced work that they have studied, and attempted to use the formula for centre of mass of a sector of a circle. Candidates generally scored either full marks or no marks for (b). Some tried to bring in the areas from (a) and ended up with dimensionally incorrect equations that earned no marks. Several chose to take moments about a different axis and then usually neglected the reaction at the pivot.